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PENDING STRESSES DUE TO TORSION IN CANTILEVER BOX BEAMS

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SUMMARY

The paper begins with a brief discussion on the origin of the bending stresses in cantilever box beams under torsion. A critical survey of existing theory is followed by a summary of design formulas; this summary is based on the most complete solution published but omits all refinements considered unnecessary at the present state of development. Strain-gage tests made by the N.A.C.A. to obtain some experimental verification of the formulas are described next. Finally, the formulas are applied to a series of box beams previously static-tested by the U.S. Army Air Corps; the results show that the bending stresses due to torsion are responsible to a large extent for the free-edge type of failure frequently experienced in these tests.

INTRODUCTION

The problem of designing a box beam in torsion is common in aircraft construction. If all cross sections of the beam are free to warp out of their plane, the walls will be in pure shear, which can be easily calculated. If, however, the cross sections are partly or completely restrained from warping, which is the case if any variation of cross section or of loading occurs along the span, then bending stresses will arise in addition to the shear stresses. These bending stresses may be very large at the root of a cantilever box attached to a rigid support; since direct bending stresses usually exist also, the calculation of bending stresses due to torsion is important. The theoretical analysis of this problem has been made fairly recently, and it is the purpose of this paper to give a critical survey of existing literature and a summary of design formulas. The reliability of these formulas was checked by some strain-gage tests, which are described and discussed. Finally, the paper shows the results of applying the formulas to a series of duralumin box beams

that had been static-tested at Wright Field by the U.S. Army Air Corps.

I. SURVEY OF EXISTING LITERATURE

The problem: Earlier attempts at solution.-- If the box beam of figure 1 is subjected to pure torque loads as indicated, the stress in the walls is pure shear and is given by the formula

$$f_s = \frac{T}{2bct} \quad (1)$$

The shear stresses cause warping of the previously plane cross sections as indicated by the dotted lines.

Suppose now the near end of the box to be built in rigidly, so that the warping cannot occur. It is clear that the support at the end, in preventing the warping, must create tensile stresses at corners 1 and 3 and compressive stresses at corners 2 and 4, decreasing linearly to zero at the center lines of the walls. The resultant of the normal stresses at the root section of a wall is a bending moment. At other sections, the stress distribution is similar but the magnitude of the stresses decreases and must be zero at the tip if the bulkheads are free to warp out of their own planes, an assumption that holds very closely for most types of bulkheads.

The distribution of the normal stresses in the vertical walls is qualitatively similar to that which would occur if these two walls acted as independent cantilever beams in bending. Similarly, the horizontal walls may be considered as a pair of beams in bending. It is therefore not surprising that the aeronautical literature records a number of attempts to solve the problem of the box by considering it as composed of two pairs of beams independent of each other. Each pair of beams was assumed to carry part of the torque, and the unknown ratio of the components was determined either by considerations of deflections or by the method of least work.

The fundamental objection to this method of solution is that the condition of continuity is grossly violated at the edges of the box. Violations of the conditions of equilibrium or continuity appear in any approximate solu-

tion but, if the solution is to have any value, these violations must be of a minor nature. If the box acted as two pairs of independent beams, adjacent edges of two beams would be in tension on one beam and in compression on the other beam, obviously a major violation of the principle of continuity. The numerous solutions of the box problem based on the method sketched in the preceding paragraph are therefore of little practical value.

Reissner's analysis.— The first correct analysis was published by Reissner (reference 1). He analyzed a rectangular box without corner flanges and assumed infinitely close spacing of the bulkheads. Writing the equation of the elastic lines of the walls and expressing all stresses in terms of the fiber stress at the edges of the box, he obtained a differential equation for this stress which he integrated for the end conditions of a root section rigidly built in and a tip section free to warp. The loading assumed was a torque distributed uniformly along the length of the box; the case of a tapered box was also treated.

The case investigated by Reissner is included as a special case in Ebner's work (reference 2) which will be later discussed.

Atkin's analysis.— Atkin (reference 3), although making a reference to Reissner's work, used an entirely different method, following the example of Timoshenko who investigated the stresses in a solid rectangular prism in torsion. With one exception, the stress distribution across a section assumed by Atkin was the same as that assumed by Reissner. For the variation along the axis, however, an arbitrary law with a free coefficient was assumed, and the coefficient was determined by the theorem of least work.

The difference in stress distribution is physically not quite clear. Mathematically, the stresses introduced should be small of the second order in comparison with the other stresses and should therefore be neglected. Furthermore, it can be shown very easily that Atkin should have modified the stress function used by Timoshenko because in Atkin's case the total strain along the length of the box does not equal the warping or, in other words, Atkin has not fulfilled the fundamental condition that the root section remain plane. Atkin's analysis is therefore of very doubtful value and comparison of numerical results

by his formula and by any other formula shows differences of several hundred percent.

Grzedzielski's analysis.— Grzedzielski (reference 4) also refers to the work of Reissner and uses a similar method. He assumes, however, that the walls of the box carry only shear and that the bending stresses are carried by flanges of area A concentrated at the corners (fig. 2). Grzedzielski's case, like that of Reissner, is contained as a special case in Ebner's derivation and will therefore not be discussed in detail. While Grzedzielski's final formula differs in form from that of Ebner, both formulas reduce to the same approximate formula for the case of very thick vertical walls (c/t_c negligible against b/t_b). The expression for the maximum normal stress at the root section becomes (with $\frac{E}{G} = 2.5$, G being the shear modulus)

$$f_n = 0.56 \frac{T}{c} \sqrt{\frac{1}{bt_b A}} \quad (2)$$

This formula overestimates the normal stresses, being based on the assumption of infinitely close spacing of the bulkheads; but it may be used as a simple approximation formula if the bulkhead spacing is close (say $a < \frac{1}{2} b$). If the factor 0.56 is reduced to 0.43, the error may be expected to be around ± 10 to 20 percent.

The case of very thick vertical walls corresponds to that of a 2-spar box. Formula (2) can therefore be obtained also as a limiting case of the 2-spar wing theory discussed in reference 5.

Ebner's analysis.— Ebner's analysis (reference 2) is considerably more comprehensive than any of the preceding ones; it includes the influence of bulkhead spacing and bulkhead rigidity. Ebner assumes the box (fig. 1) to be broken up into component boxes by cuts at the bulkheads; each component, or "cell" (fig. 3), is then loaded by two torque loads that can be computed from the externally applied torque, by intermediate torque loads applied between the bulkheads (not shown in fig. 3, because they do not always exist), and by two groups of antisymmetrical forces X_R that are caused by the adjoining cells which partly prevent the warping of the cross sections. By means of the principle of consistent deformations, a system of equa-

tions for the X forces at the bulkheads can be derived. This method of analysis permits the calculation of boxes with any variation of dimensions or loading along the axis.

The design formulas given in the next section are either taken directly from reference 2 or obtained by simple mathematical approximations from formulas given there.

II. SUMMARY OF DESIGN FORMULAS

Simplifying Assumptions

In view of the large uncertainties attending the calculation of built-up structures of thin sheet, it is considered sufficient to give only the most important formulas. The following assumptions and simplifications are made.

(1) As far as bending stresses due to torsion are concerned, it is generally sufficient to consider only the first bay at the root, or perhaps the first two, because the decrease of the stresses along the axis is very rapid (roughly following an e^{-x} curve). On the basis of his numerical calculations, Ebner considers this simplification as applicable to most practical cases; calculations on 2-spar wings with stressed-skin covering (reference 5) tend to support his viewpoint.

(2) The bulkheads are assumed to be rigid in their own plane (but free to warp out of their plane). Errors due to this assumption will probably be less than about 5 percent if the bulkheads are solid sheets or trusses but may become very much larger if the bulkheads are sheets with large lightening holes or frames.

(3) The torque moment at the root is used regardless of the type of torque distribution along the axis. The formulas given are those for a torque moment concentrated at the tip; they were chosen on account of their greater simplicity in spite of the fact that the most usual case is probably that of distributed torque. Again judging by Ebner's and other numerical calculations, the error due to using the tip-torque formula in the case of a uniformly distributed torque is not likely to exceed 20 percent if the length of the box is more than three times the width

and the bulkhead spacing is less than the width. In view of the fact that tests of thin-sheet structures can hardly be duplicated within 20 percent and that the bending stresses due to torsion are generally only a part of the total stresses, this error does not appear to be excessive.

(4) In the case of a box with dimensions varying along the span (box tapering or wall thicknesses changing), good approximations are obtained, according to Ebner, if it is assumed that all the cells of the box have the same dimensions as the cell at the root.

If it appears desirable or necessary to obtain a higher degree of accuracy than the following formulas will afford, recourse must be had to the formulas and methods developed in reference 2.

Case A. Walls of Box Do Not Buckle

The fundamental case is the case of a box with walls heavy enough to withstand buckling until the design load is reached. For such a box, the force X_R at the root (fig. 3) is given by

$$X_R = \frac{\eta}{\sqrt{3\rho} (1 + \rho/4)} \frac{a}{b c} \tau \quad (3)$$

and the normal stress at the root is given by

$$f_n = \frac{6X_R}{bt_b + ct_c + 6A} \quad (4)$$

The force X_B at the first bulkhead outboard of the root is given approximately by

$$X_B = X_R e^{-\varphi} \quad (5)$$

The variation of f_n between bulkheads is linear. In these formulas

$$\eta = \frac{b/t_b - c/t_c}{b/t_b + c/t_c} \quad (6)$$

$$\rho = \frac{16a^3 G/E}{(b/t_b + c/t_c) (bt_b + ct_c + 6A)} \quad (7)$$

$$\phi = \cosh^{-1} \left| \frac{1 + \rho}{1 - \frac{\rho}{2}} \right| \quad (8)$$

The positive directions of T and X are those shown in figure 3. It is important to note that the sign of X depends on the sign of η , so that b and c must be in the same position relative to T and X as shown in figure 3. The rule of signs may be stated as follows: The normal stresses in the pair of walls with the smaller section aspect ratio (depth to thickness) are of the same sign as if these walls acted as two independent cantilever beams in bending.

For a certain range, formulas (3) and (5) can be approximated by

$$X_R = \frac{\eta}{1 + \rho} \frac{a}{b c} T \quad (3a)$$

$$X_B = \frac{X_R (1 - \frac{\rho}{2})}{2 (1 + \rho)} \quad (5a)$$

The error in formulas (3a) and (5a) is less than 1 percent if $1.5 < \rho < 3$ and less than 5 percent if $0.9 < \rho < 6$.

Two special cases are possible where ρ becomes zero. The formulas are given here because they cannot be obtained by simply substituting $\rho = 0$ in formula (3).

If the thickness of one pair of walls, say of the horizontal walls b , becomes zero then

$$X_R = \frac{l}{b c} T \quad (3b)$$

which is the case of two independent spars with concentrated loads at the tip.

If the bulkhead spacing becomes infinitely close

$$X_R = \frac{\eta T}{K bc} \quad (3c)$$

where K is defined by

$$K^2 = \frac{48 G/E}{(b/t_b + c/t_c) (bt_b + ct_c + 6A)}$$

The shearing stresses at the root are given for the wide wall b by

$$f_s = \frac{T}{2 bc t_b} - \frac{1}{2a t_b} (X_R - X_B) \quad (9)$$

and for the narrow wall c by

$$f_s = \frac{T}{2 bc t_c} + \frac{1}{2a t_c} (X_R - X_B) \quad (10)$$

under the assumption that the shear stress is uniformly distributed over the depth of each wall.

In formulas (9) and (10), the first term is the shearing stress for pure torsion (i.e., with all sections free to warp), and the second term gives the additional shearing stress accompanying the bending stresses. The wide wall is relieved of some shear; the narrow wall has its shear increased by the restraining action of the support.

At the first bulkhead, the shear stresses may be computed by using formulas (9) and (10), substituting X_B for $(X_R - X_B)$.

Case B. All Walls Form Diagonal-Tension Fields

If all four walls of the box form diagonal-tension fields, the force X_R is given by

$$X_R = \frac{\gamma}{\sqrt{\alpha^2 - 1}} \frac{a}{b c} T \quad (11)$$

$$\text{where } \gamma = \frac{\eta + \frac{b+c}{b-c} \rho_2}{1 - \frac{1}{2} \rho_1 + \rho_2} \quad (12)$$

$$\alpha = \frac{1 + \rho_1 + \rho_2}{1 - \frac{1}{2} \rho_1 + \rho_2} \quad (13)$$

$$\rho_1 = \frac{2/3 a^2}{(b/t_b + c/t_c) A} \quad (14)$$

$$\rho_2 = \frac{1/8 (b - c)^2}{(b/t_b + c/t_c) A} \quad (15)$$

The total force acting on a flange is the sum of the force preventing warping and of the force due to the diagonal tension in the webs,

$$F_R = \pm [X_R + \frac{b - c}{4a} (X_R - X_B)] - \frac{b + c}{4bc} T \quad (16)$$

The diagonal-tension stress is in wall b

$$f_t = \frac{T}{bc t_b} - \frac{1}{a t_b} (X_R - X_B) \quad (17a)$$

and in wall c

$$f_t = \frac{T}{bc t_c} + \frac{1}{a t_c} (X_R - X_B) \quad (17b)$$

Case C. The Cover Walls (b) Form Diagonal-Tension Fields

The case where the cover walls form diagonal-tension fields, probably the most common one, could be obtained by combining the methods used for solving cases A and B. The following section will, however, discuss some reasons for doubting that the assumptions used for case B are in good agreement with the physical facts. The writer therefore suggests the use of the formulas for case A with the following modifications, which he believes to give a picture closer to the true physical conditions.

(1) In the terms b/t_b of formulas (6) and (7), the thickness t of the buckled wall is replaced by the effective thickness in shear of a sheet in diagonal tension

$$t_e = 1/4 \frac{E}{G} t \quad (18)$$

or $t_e = 5/8t$ for metal.

(2) In formulas (4) and (7) the term $(bt_b + ct_c + 6A)$ is replaced by $6A_e$, where A_e is the effective flange area defined by

$$A_e = A + 1/6 ct_c + wt_b \quad (19)$$

In this formula A is the actual concentrated flange area (if existing), and w is the effective width of a thin flat sheet in compression ($w = 15t_b$ for duralumin).

(3) To the normal stress in the flange computed by means of (4), which is tensile or compressive, depending on X , is added algebraically the compressive stress due to diagonal tension in the cover

$$f_n = - \frac{T}{4c A_e} \quad (20)$$

Remarks

The formulas for case B are based on the elementary theory of the Wagner beam assuming, among other things, that the struts are rigid, closely spaced, and so well connected to the web that the diagonal-tension folds are interrupted at the struts and that the diagonal-tension stress is uniform in a field between struts. Actually, it is likely that the conditions will approach the opposite extreme of struts not connected to the web so that the tensile stress will be constant along any given fold from flange to flange. This discrepancy is of small importance in the design of ordinary Wagner beams but, in the case under consideration here, the flange stresses vary from their maximum value at the root to practically zero in a distance comparable with the length of a tension fold. This fact, together with the consideration that in sheet-metal construction local deviations from the assumptions of the theory are often large and unpredictable, leads to the conclusion that the formulas for case B cannot be expected to give very close agreement with facts. The very scanty quantitative information available seems to indicate that the formulas give excessively conservative (high) stresses in the flanges.

Another factor that should be considered in some

cases is the fact that the transition of a flat sheet from a state of shear to a state of pure diagonal tension is very gradual. Qualitative support of this claim is given by various test reports; the only quantitative information is contained in reference 6, which describes strain-gage tests on a Wagner beam. The information is not very exact, because the stress in the flange due to the diagonal tension in the web is only a small part of the total flange stress; using this information as long as nothing else is available, it may be concluded that the diagonal-tension state has been reached, practically speaking, when the shear load exceeds the buckling shear 25 to 50 times (based on the measurements in tension and compression flanges, respectively). The main effect of this difference is that the compression term in (16) and the compression computed by (20) are reduced; it is suggested that these terms be multiplied by the ratio (actual shear/(25 X buckling shear)). This procedure was applied to the two tests discussed in Part III, in which the cover buckled, and was found to be conservative.

It might not be amiss to point out that box beams expected to work partly or entirely in diagonal tension require, theoretically at least, distinct flanges at the corners. Actually, it is quite possible to do without such flanges (beam 4 of Part IV) because the corners of a box have some stiffness and because, as pointed out, the sheet continues to work partly in shear after buckling. In beams of this type, however, only very rough estimates of the stresses can be made because most assumptions of the theory are no longer valid.

It might also be pointed out that the formulas for Case A may be applied to boxes with trussed walls, such as wing frames with double drag bracing or fuselages. It is only necessary to imagine the diagonals of the trusses replaced by solid sheet webs of such thicknesses as to give the same shear deflection to the trusses. The method should, however, be used with caution if there are large irregularities in the structure, which will be often true of fuselages.

III. STRAIN-GAGE TESTS ON THREE BOX BEAMS

In order to gain some idea of how closely the theoretical stresses may be actually approached, strain-gage tests were made by the N.A.C.A. on three duralumin boxes. The di-

mensions of these boxes are shown in figure 4, which also shows the three gage locations used. The bulkhead at the root was made of 2-inch steel to prevent any possibility of warping. The boxes were assembled by means of screws instead of rivets to save manufacturing new side channels and bulkheads for each box. The strain gages used were Tuckerman gages (reference 7). Two gages of 2-inch gage length were used in the locations near the root section and two 1-inch gages at the first bulkhead. The load applied was a pure torque exerted on a loading arm at the tip by two weights, one acting down and one acting up over a pulley. The pulley friction was less than $3/4$ percent for the pulley used for the 0.011-inch box. For the pulley used for the 0.022-inch and 0.044-inch boxes it was about 5 percent and was taken care of by additional weights.

The gages were located symmetrically with respect to the axis, so that there were always two 2-inch gages and two 1-inch gages at corresponding positions on the tension flange and on the compression flange. After taking a set of readings, the torque was reversed. In preliminary tests it was found necessary sometimes after reversing the torque to load the box several times before successive sets of readings agreed. In all later tests the box was therefore preloaded three times to about two thirds of the maximum load used in the test before a regular test run was made. In a number of cases the box was also turned over and the tests were repeated on the opposite side.

The accuracy of reading the 2-inch gages is somewhat less than ± 20 pounds per square inch. The accuracy of reading the 1-inch gage should be ± 40 pounds per square inch but, owing to difficulties in reading, it was quite often only about ± 80 pounds per square inch. Successive test runs with the gages left in their locations practically always agreed within the limit of accuracy of reading. All readings were taken 3 minutes after applying the load increment.

Figures 5, 6, and 7 show the test results. It will be noted that, in general, tension and compression were averaged separately, but in some cases the differences between the average tension and the average compression were too small to show, so that only the normal stress (average of tension and compression) was plotted. Figures 6 and 7 also indicate the difference between tension and compression flange, i.e., twice the compressive stress due to diagonal tension computed by (20) and by the procedure rec-

ommended in Part II. The buckling load of the cover sheet is indicated in figures 6 and 7. In figure 5 it falls outside the range plotted.

Figure 8 shows the average normal stress in the 0.011-inch box, taken from figure 7, compared with the stresses calculated for the cover not buckled and buckled. Near the root, the cover was stiffened by the 2-inch bulkhead so that the buckles were hardly perceptible and this result shows up in the stress curve, which follows first the line calculated for the cover not buckled and then gradually bends over to the line calculated for a buckled cover. At the 1-5/8-inch station, the experimental stress curve is between the two calculated lines; at the first bulkhead, the actual stress is considerably higher than the calculated stress and deviates considerably from the straight-line law.

Figure 9 shows for all three boxes the observed and the calculated stresses plotted against their spanwise location. It will be seen that in the bay the actual stresses for the 0.022-inch box were considerably lower than calculated, and the difference is even larger for the 0.044-inch box. At the first bulkhead, however, the observed stresses are considerably larger than those calculated for all three boxes.

It has been pointed out that the accuracy of the stress measurements at the first bulkhead was very much lower than at the other stations. The measurements at the stations 1/2 inch and 15/16 inch from the root are somewhat doubtful because it was necessary to remove the outside screws in order to permit installation of the gages; this change may have permitted the stress trajectories to curve away from the edge of the box. The 1-5/8-inch stations are therefore the most important ones and are used for comparison.

The differences between observed and calculated stresses may be explained in part by the assumption that the efficiency of the bulkheads decreased with increase of cover thickness. If the bulkhead efficiency is defined as the ratio of the actual bulkhead spacing to the "effective" bulkhead spacing that gives agreement between calculated and observed stresses at the 1-5/8-inch station, the efficiencies are about 100, 45, and 30 percent for the 0.011-inch box, the 0.022-inch box, and the 0.044-inch

box, respectively. Figure 9 shows the stresses calculated with the corresponding "effective" bulkhead spacings (dotted lines), which give agreement at the 1-5/8-inch station and decrease the discrepancy at the first bulkhead. Even with the fictitious bulkhead spacing, however, the actual stress at the first bulkhead is 2-1/2 to 3 times the calculated stress.

On the basis of these tests only the tentative conclusion may be drawn that, in relatively thick-walled box beams, the maximum stresses will be considerably lower than calculated stresses unless liberal allowances for bulkhead inefficiency are made because it is not likely that practical bulkhead constructions will be more efficient than those used in these tests. In boxes with very thin covers, however, the theoretical stresses at the root may be reached or even exceeded. The rate of decrease of stress along the span appears to be always much slower than calculated, a fact that should be borne in mind when investigating sections outboard of the root. No definite recommendations can be made in this respect until a more detailed experimental investigation is made, using relatively closer bulkhead spacings and larger boxes than used in these tests.

IV. APPLICATION TO WRIGHT FIELD TEST BEAMS

References 8 and 9 describe a series of torsion tests on box beams. Six beams of this series were rectangular duralumin boxes of the same construction as shown in figure 4; table I gives the dimensions.

The test set-up is indicated in figure 10. The beams were tested with a constant bending load P and an increasing torque until "failure" occurred. It is evident that the central section of the box is under the same condition as the root of a cantilever box.

The cover buckled in all tests, but the side walls in only one test (beam 4). The bending stiffnesses of the beams were therefore calculated under the assumption that on the compression side the area A and an effective width $w = 15t$ of the cover plate were active. Experimental stiffnesses given in the test report were in most cases only very slightly larger than the computed ones; consequently, it was considered sufficiently accurate for

the purpose of calculating the bending stresses due to P to compute the section modulus on the assumption of $w = 15t$ and make a correction depending on the experimental and calculated bending stiffnesses. Outside of this correction, the stresses were calculated in accordance with the recommendations made in Part II. The results of the calculations are shown in table II.

The test log states definitely that beams 2, 3, and 6 failed by buckling of the free edge between rivets. The rivet spacing was 1 inch; the Euler stress ($C = 1$) for the thicker component of the free edge was calculated, and the last column of table II gives the fixity coefficient developed in the tests, i.e., the ratio of total stress to Euler stress. (The Euler stress of the thicker component was used because it was considered that the thinner component cannot buckle until the thick one buckles, the rivets preventing the shortening of the chord necessary for buckling.)

It will be noted that, contrary to the usual assumption of $C = 4$ for this type of buckling, the coefficients developed were between 1.3 and 1.8, excepting beam 6 with $C = 2.4$. This last coefficient is high because the calculation of the stresses neglected the fact that the cover sheet had a mahogany veneer sheet cemented to it, which helped to reduce both the direct bending stresses and the bending stresses due to torsion.

No statement is made in the test log of the type of failure for beam 4. Considering the unusual construction of this beam it is probable that excessive deformation of the whole beam or of a local zone determined the limiting load. Beam 5 was stated to have "developed a bad buckle near center." The fixity coefficients calculated for these two beams are therefore of interest only insofar as they show that the calculated values are not unreasonable.

The report of tests on these and other box beams stresses the importance and prevalence of free-edge failure. Table II shows that the bending stresses due to torsion are decisive factors in this type of failure.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., April 10, 1935.

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TABLE I

Dimensions of Test Beams

Beam	a	b	c	t _b	t _c	A
1	16	12	4	0.012	0.049	0.0305
2	12	12	4	.012	.049	.0305
3	8	12	4	.012	.049	.0305
4	12	12	4	.040	.035	.0525
5	12	12	4	.021	.041	.0434
6	12	12	4	.010	.035	.0282

Dimensions are in inches.

TABLE II

Calculated Stresses in Test Beams

Beam	f ₁	f ₂	f ₃	f _{total}	C
1	10,240	10,200	7,370	27,810	1.29
2	9,170	11,600	7,680	28,450	1.32
3	7,990	15,420	9,400	32,810	1.52
4	4,640	12,440	2,880	19,960	1.80
5	9,030	8,770	6,470	24,270	1.60
6	13,200	7,720	5,270	26,190	2.37

Stresses are in pounds per square inch.

f₁, normal stress due to bending load.

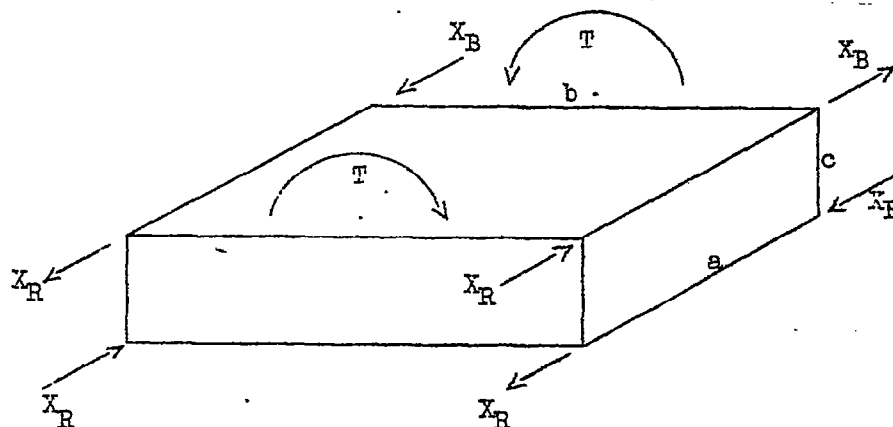
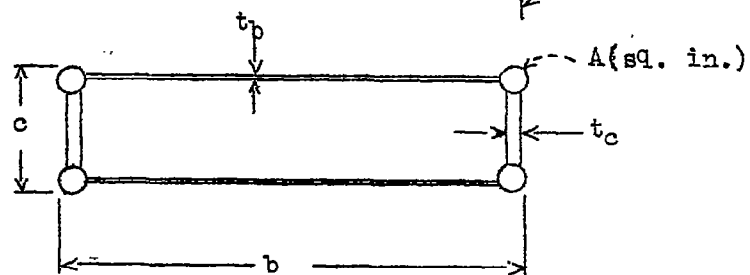
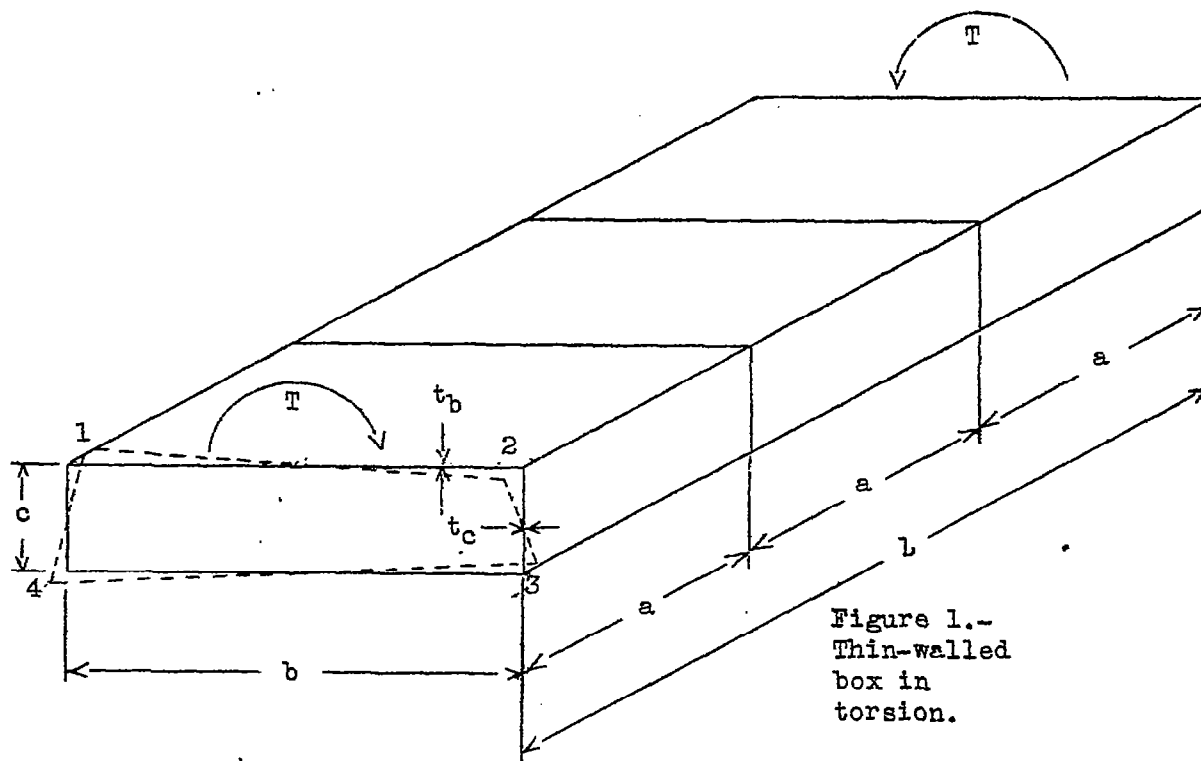
f₂, normal stress due to torsion.

f₃, normal stress due to diagonal tension.

$$f_{\text{total}} = f_1 + f_2 + f_3$$

$$C = f_{\text{total}} / f_{\text{Euler}}$$

$$f_{\text{Euler}} = \frac{\pi^2 E t^2}{12 (1 - \mu^2) b^2} \sim 0.90 E \left(\frac{t}{b}\right)^2$$



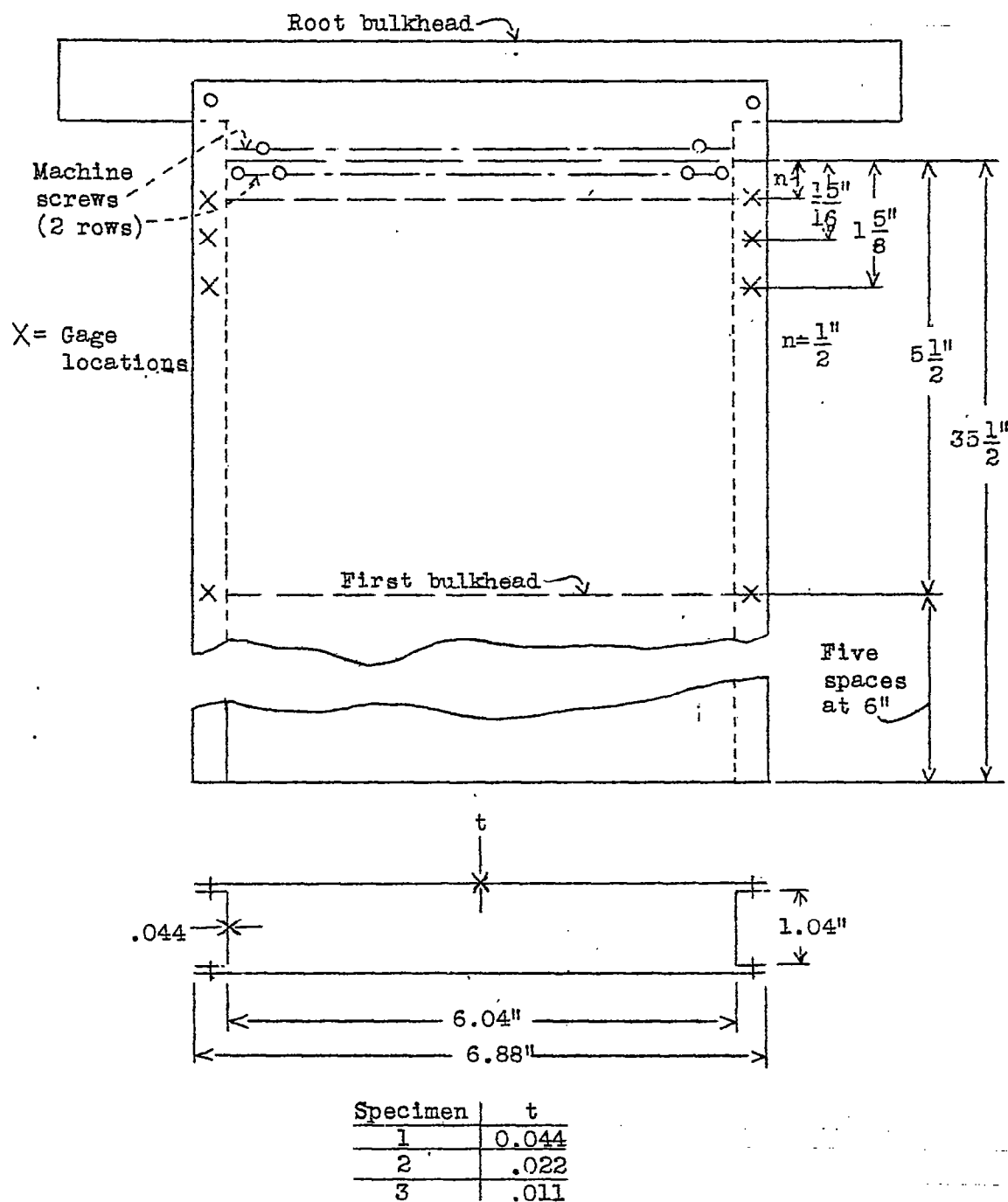


Figure 4.-Test specimens for strain gage tests.

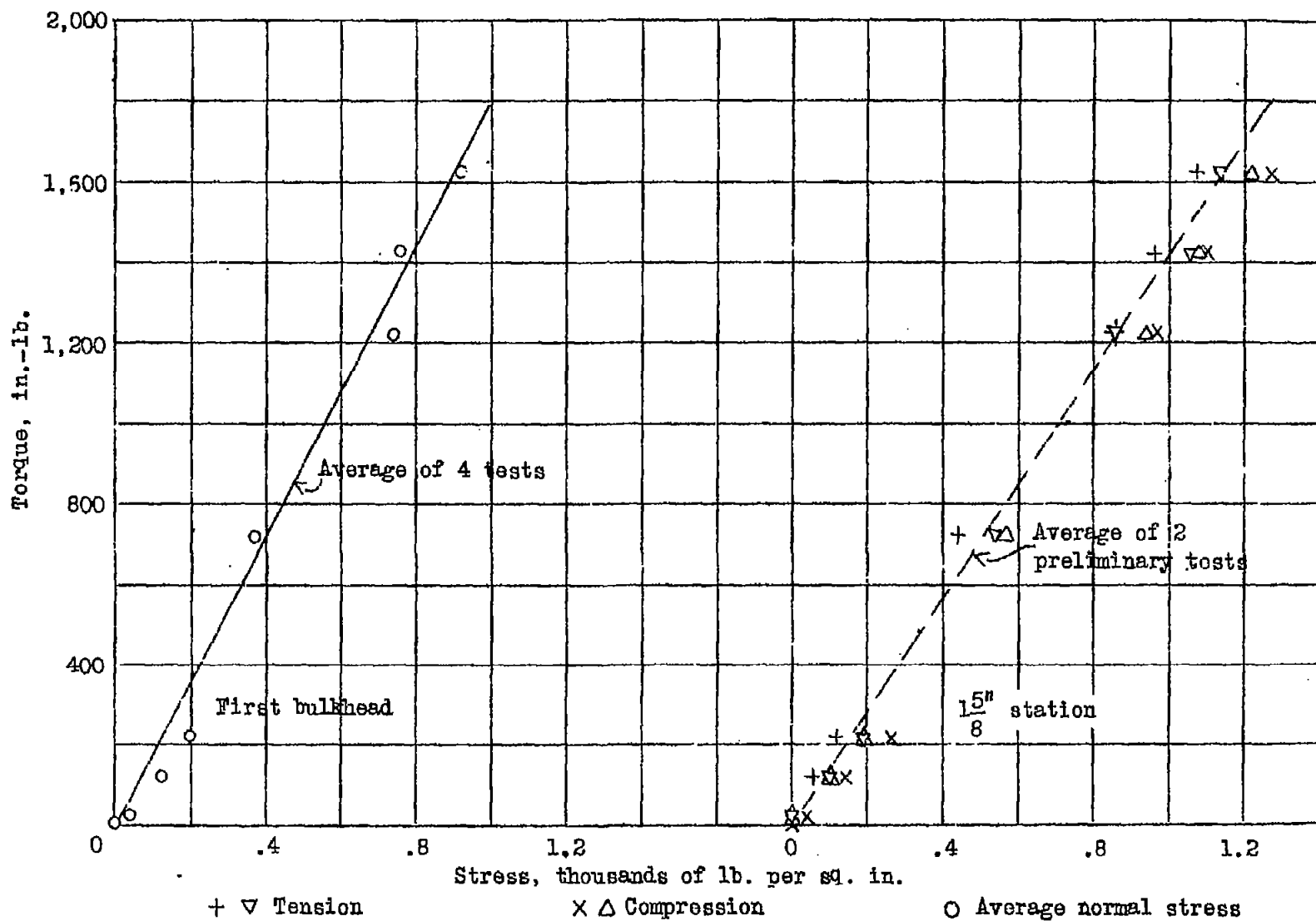


Figure 5.-Load-stress curves for 0.044 inch box.

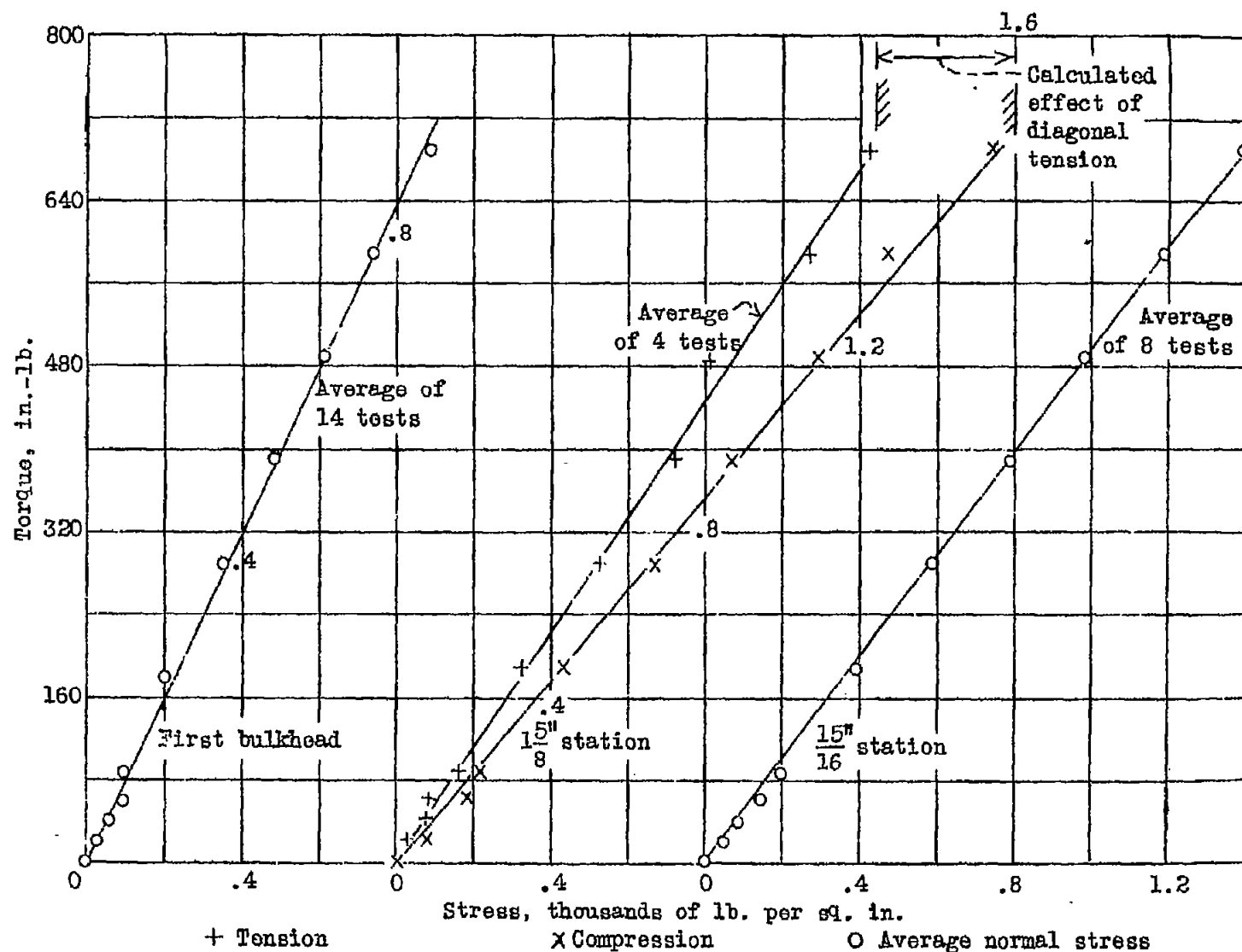


Figure 6.-Load-stress curves for 0.022 inch box.

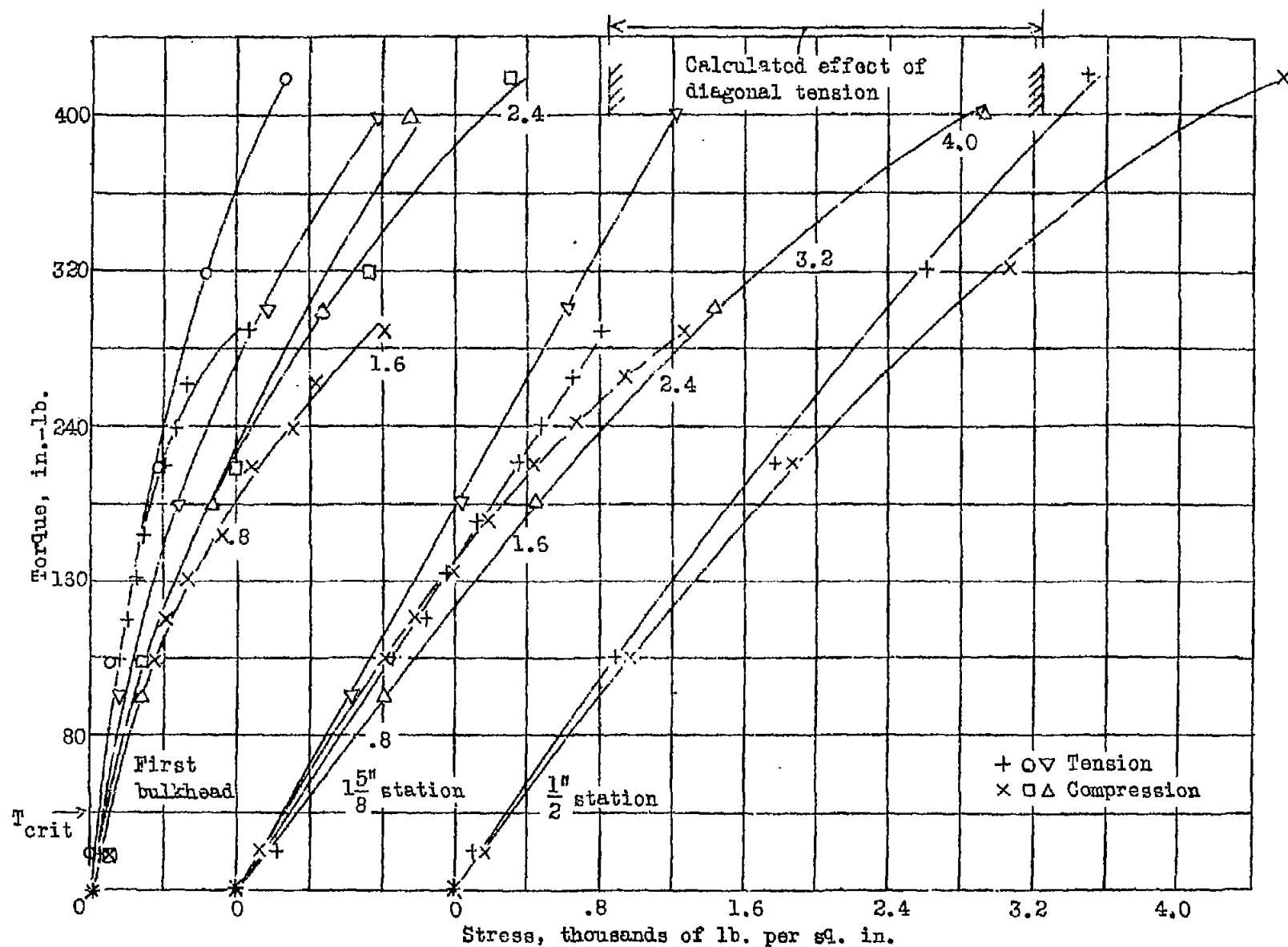


Figure 7.-Load-stress curves for 0.011 inch box.

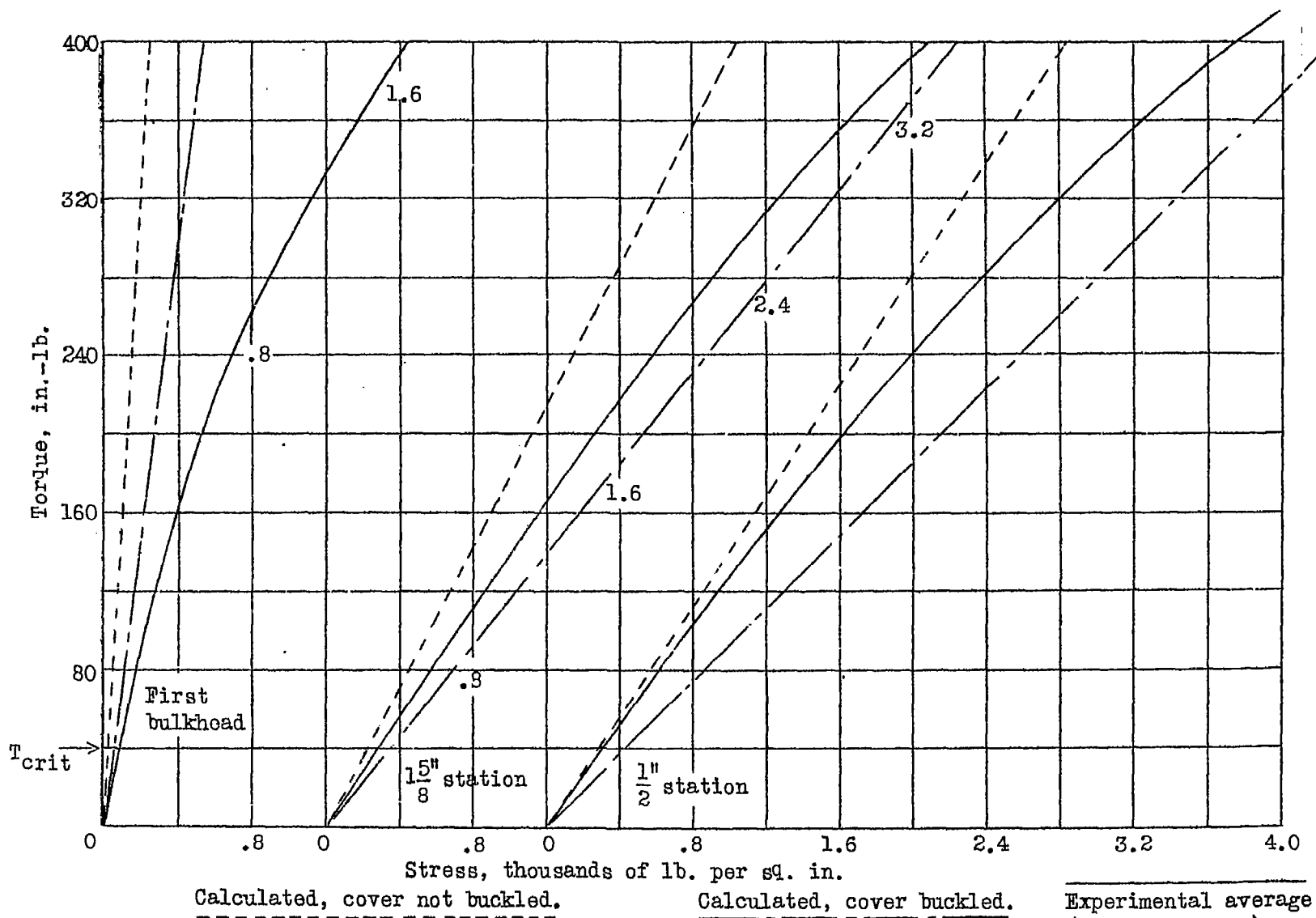


Figure 8.—Comparison between calculated and experimental load-stress curves for 0.011 inch box.

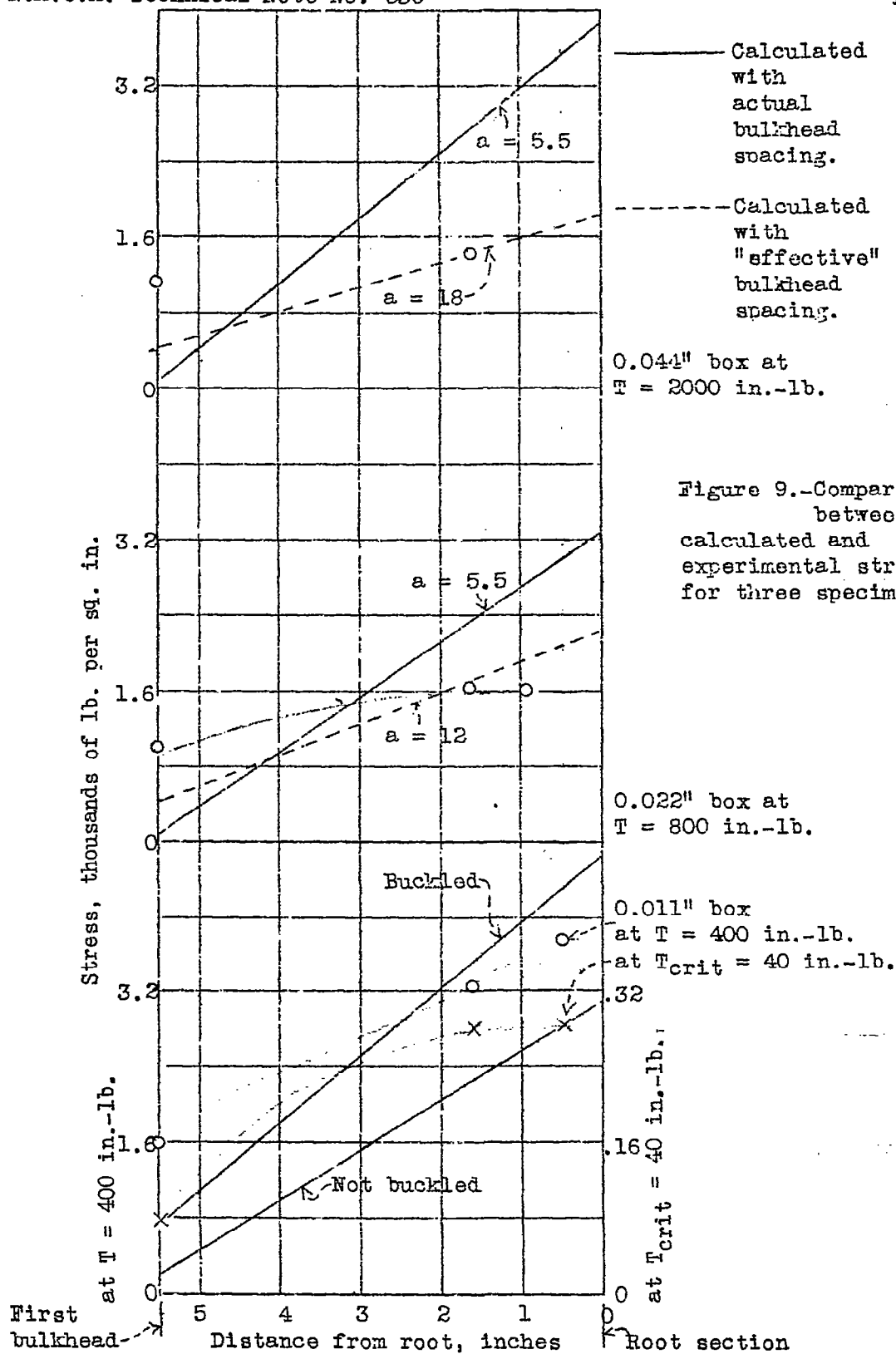


Figure 9.-Comparison
between
calculated and
experimental stresses
for three specimens.

0.022" box at
T = 800 in.-lb.

0.011" box
at $T = 400$ in.-lb.
at $T_{crit} = 40$ in.-lb.

.32

0.16 at $T_{crit} = 40 \text{ in.-lb.}$

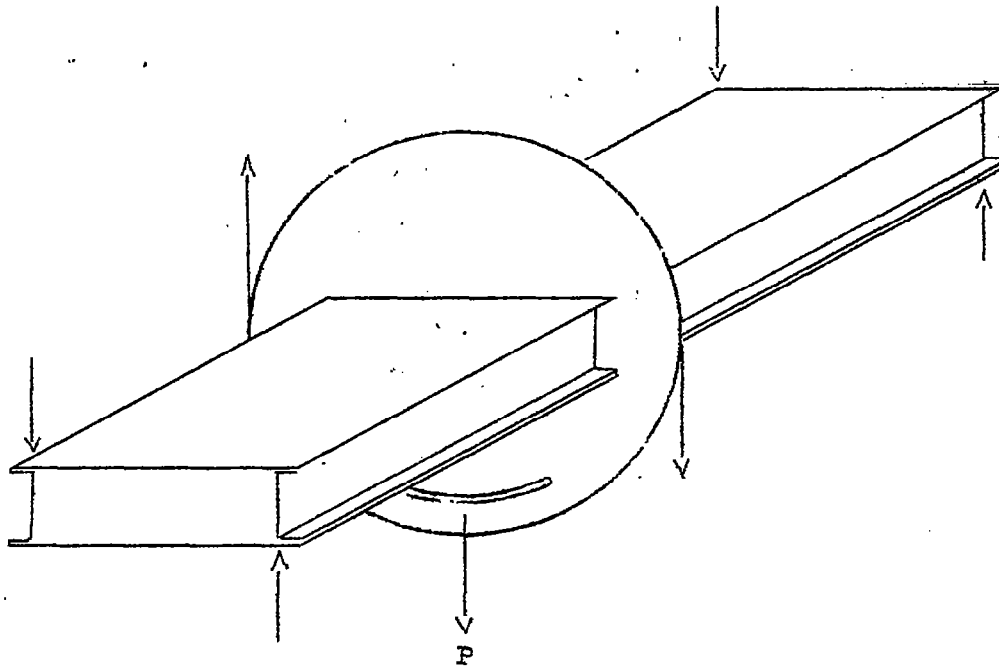


Figure 10.-Set-up for static tests.